

Estimation and initialization of quantum network via continuous measurement on single node

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Abstract—For reliable and consistent quantum information processing carried out on a quantum network, the network structure must be fully known and a desired initial state has to be prepared on it. In this paper, we consider a spin network with only a single node accessible and provide continuous-measurement-based methods to fulfill the above two requirements. More specifically, the measurement results continuously obtained are used to update the estimate of the network structure, which converges to the true structure with high probability; for the latter problem, we further employ an adaptive measurement scheme to deterministically drive the state to a desired target state for network initialization.

I. INTRODUCTION

Quantum information processing can be usually performed on a highly networked system composed of a large number of subsystems [1], [2]. A specifically useful result is that we can achieve universal quantum computation by locally controlling all those subsystems [3], [4], [5], [6]. However, accessibility to all subsystems is impractical, in the sense that the cost for actuators increases with the number of subsystems. Also manipulation error occurred at each subsystem will be accumulated, which as a result can largely degrade the performance of information processing. These issues thus stimulate development of a network control method based on manipulating only a small set of the subsystems, whereas the other subset will be controlled indirectly through interactions among the network. Actually some advantages as well as limitations of this approach in various applications to quantum information have been studied; for instance, state preparation [7], purification [8], [9], state transfer [10], parameter estimation [11], [12], and universal computation [13], [14]. Note that, when a solid system with its surface only accessible is employed to construct the network, the local accessibility is merely a constraint rather than a control strategy.

Now let us turn our attention to some requirements imposed on a network for quantum information processing. In particular, the following two are critical; (1) the dynamical behavior of the network must be fully known, and (2) a desired initial state of the network must be prepared. Together with the fact mentioned in the first paragraph, we are then reasonably motivated to develop a scheme that allows us to access to only a part of the network and yet to achieve the above two goals (1) and (2). Actually, Refs. [11], [12] and [8], [9] both have considered a specific network with limited access, and they provided a method for estimating the

parameters of the network and that for stabilizing a certain entangled state, respectively. In this paper, we especially consider spin networks with only a single node accessible and then provide continuous-measurement-based approaches to achieve the two goals. Continuous measurement is no more than the repetition of Bayesian update of the system state based on the measurement result, and thus it can suitably be applied to the problems of parameter estimation [15], [16], [17] and feedback control for state preparation [18], [19], [20].

We now describe our results. First, towards the goal (1) we particularly provide an algorithm to test whether any given pair of nodes of the network are connected or not; that is, it estimates the network structure using continuous measurement performed only on a single spin. We will demonstrate in numerical simulations that our scheme correctly identifies the network structure with high success probability. Note that our problem is different from that studied in [11], [12], in the sense that they have tackled the problem of parameter estimation of a certain network with known structure. We are rather motivated by the fact that sometimes the network structure itself is unknown; for instance, in the case of solid systems, subsystems are served by atoms produced at different sites possibly randomly, implying the need to estimate the network structure. It is also worth to note that this kind of structure estimation problem often appears in classical regime, e.g., reconstruction of the network structure of gene mRNA concentrations [21] and estimation of relationships in social networks [22], [23].

Next, we consider the problem of deterministic stabilization of the spin states polarizing along a common direction, i.e., $|\phi, \phi, \dots, \phi\rangle$, where $|\phi\rangle$ is a desired spin state at each site; this *spin coherent state* is a most typical initial state required for quantum information processing in a network. As in the previous case, we use continuous measurement performed only on a single spin. As shown in [8], [9], any fixed measurement can stabilize a given target state only probabilistically; but we provide a deterministic method based on the so-called *adaptive measurement* technique [24], [25], which is a kind of feedback control scheme that changes the measured observable continuously in time, depending on the past measurement results.

Notation. The dagger \dagger denotes the Hermitian conjugate of a matrix. \otimes denotes the Kronecker product of two matrices. A *pure quantum state* is represented by a vector, which is conventionally written in terms of “ket” as $|\phi\rangle$. Especially, $|0\rangle = [1, 0]^T$ and $|1\rangle = [0, 1]^T$ represent the state of spin-up and spin-down, respectively. Also we use $|\phi, \phi, \dots, \phi\rangle =$

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$|\phi\rangle \otimes |\phi\rangle \otimes \dots \otimes |\phi\rangle$ for simplicity. A more general *mixed state* is represented by a non-negative Hermitian matrix $\rho = \rho^\dagger \geq 0$; a pure state is a special class of a mixed state, and it can be expressed as $\rho = |\phi\rangle\langle\phi|$. I_n is the $n \times n$ identity matrix.

$$\sigma^x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

are the Pauli matrices.

II. NETWORK STRUCTURE ESTIMATION VIA CONTINUOUS MEASUREMENT ON SINGLE NODE

A. The estimator

We first describe the general setup of continuous measurement. This can be physically realized by coupling an optical probe field to the system of interest and measuring the output field continuously in time. In particular when a homodyne detector is used for measurement, the time evolution of the system state $\rho_t = \rho_t^\dagger \geq 0$ conditioned on the measurement results $\mathcal{Y}_t = \{Y_s \mid 0 \leq s \leq t\}$ is given by the following *stochastic master equation* (SME) [20], [26], [27]:

$$d\rho_t = -i[H, \rho_t]dt + \gamma \mathcal{D}[c]\rho_t dt + \sqrt{\gamma} \mathcal{H}[c]\rho_t dW_t, \quad (1a)$$

$$dY_t = \sqrt{\gamma} \text{Tr}[(c + c^\dagger)\rho_t]dt + dW_t, \quad (1b)$$

where H is the system Hamiltonian, c the coupling operator between the system and the probe field, γ the coupling strength, and dW_t the standard Wiener increment with mean zero and variance dt . Also we have defined

$$\begin{aligned} \mathcal{D}[c]\rho &= c\rho c^\dagger - \frac{1}{2}(c^\dagger c\rho + \rho c^\dagger c), \\ \mathcal{H}[c]\rho &= c\rho + \rho c^\dagger - \text{Tr}[(c + c^\dagger)\rho]\rho. \end{aligned}$$

Note that Eq. (1) is a quantum counterpart to the classical Kushner-Stratonovich equation that describes the time evolution of a conditional probability density. Therefore, as in the classical case, the conditional expectation $\text{Tr}(A\rho)$ represents the least mean squared error estimate of an observable A .

In our case, the system is an N -spin network whose structure is captured by the graph G with the set of nodes (vertices) $V(G)$ and that of edges $E(G)$; that is, each node represents a single spin state $|\phi\rangle = [a, b]^\top$ and $E(G)$ represents the set of pair of spins connected with each other. Since each spin is of dimension 2, the whole network is a 2^N -dimensional large system. Especially we assume that the interaction between the nodes occurs through the XY coupling Hamiltonian [10]:

$$H = \sum_{(j,k) \in E(G)} \lambda(\sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y), \quad (2)$$

where σ_j^x , σ_j^y , and σ_j^z are the Pauli matrices acting on the j th spin; thus the notation means e.g. $\sigma_j^x = I_2 \otimes \dots \otimes \sigma_j^x \otimes \dots \otimes I_2$. Here for simplicity the coupling constants are assumed to be known and uniform. Next, suppose that only the first spin is accessible for manipulation; hence we are allowed to continuously measure only the first spin, in which case the coupling operator c in Eq. (1) is given by

$$c = \sigma_1^z \otimes I_{2^{N-1}}. \quad (3)$$

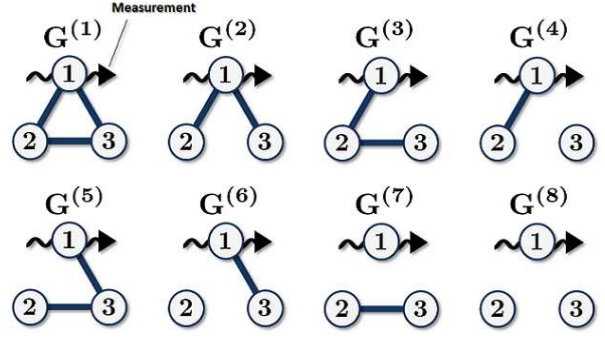


Fig. 1. Possible graph structures in the three spin network. In each case the first node is measured, as indicated by the wavy arrow.

As a result, the state of the network with graph G , whose first spin is continuously measured, is subjected to the SME (1) with the Hamiltonian (2) and the coupling operator (3).

Now, let us consider the situation where the network structure G is unknown; then the problem is to estimate the network structure by determining which spins are connected. This is equivalent to choosing the *true graph* $G^{(i_0)}$ from all possible *nominal graphs* $G^{(1)}, \dots, G^{(m)}$, where $m = 2^{N C_2}$ is the number of all combinations of the edges in the N -spin network. For a network composed of three spins, for instance, we have totally eight candidates of graphs $G^{(1)}, \dots, G^{(8)}$ whose edges are respectively given by $E(G^{(1)}) = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$, $E(G^{(2)}) = \{\{1, 2\}, \{1, 3\}\}$, $E(G^{(3)}) = \{\{1, 2\}, \{2, 3\}\}$, \dots and $E(G^{(8)}) = \{\}$, as shown in Fig. 1.

To solve the problem, we employ the idea developed in [16], [17]. Define a discrete probability distribution $\{p^{(1)}, \dots, p^{(m)}\}$ with $p^{(i)}$ denoting the probability such that $G^{(i)}$ is likely the true graph of the network. Let us further denote $\rho^{(i)}$ the system state corresponding to the i th nominal graph $G^{(i)}$. Then, the state of the enlarged system composed of the classical index set and the nominal quantum systems is represented as

$$\rho_t^E = \sum_{i=1}^m p_t^{(i)} |\psi_i\rangle\langle\psi_i| \otimes \rho_t^{(i)}, \quad (4)$$

where $\{|\psi_i\rangle\}$ is the set of m -dimensional orthonormal vectors with the index i corresponding to the i th graph. Now, the system with graph $G^{(i)}$ is driven by the Hamiltonian

$$H^{(i)} = \sum_{(j,k) \in E(G^{(i)})} \lambda(\sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y), \quad (5)$$

while the coupling operator (3) is commonly taken for all nominal graphs. That is, the enlarged system is driven by the Hamiltonian

$$H^E = \sum_{i=1}^m |\psi_i\rangle\langle\psi_i| \otimes H^{(i)} \quad (6)$$

and is measured continuously through the coupling operator

$$c^E = I_m \otimes c \quad (7)$$

with c given in Eq. (3). Substituting Eqs. (4), (6), and (7) for the SME (1), we have

$$dp_t^{(i)} = -i[H^{(i)}, \rho_t^{(i)}]dt + \gamma \mathcal{D}[c]\rho_t^{(i)}dt + \sqrt{\gamma} \mathcal{H}[c]\rho_t^{(i)}dW'_t, \quad (8a)$$

$$dp_t^{(i)} = 2\sqrt{\gamma} \{ \text{Tr}(c\rho_t^{(i)}) - \text{Tr}(c^E \rho_t^E) \} p_t^{(i)} dW'_t, \quad (8b)$$

$$dW'_t = dY_t - 2\sqrt{\gamma} \text{Tr}(c^E \rho_t^E)dt, \quad (8c)$$

where Y_t is the measurement result; note that Y_t is the output of the true system having the Hamiltonian $H = H^{(i_0)}$ and the coupling operator (3). We recursively calculate Eq. (8) to update the probability distribution $p^{(i)}$ as well as the state $\rho^{(i)}$, using the measurement result Y_t ; we then expect that, after many iterations, $p^{(i_0)}$ will get the maximum value (recall that i_0 is the index representing the true graph).

B. Example 1: three spin case

Let us consider the simple network composed of three nodes; in this case, as depicted in Fig. 1, we have $m = 8$ candidates as the graph structure. For demonstration, we here set the true system as the chain-type network, i.e., $G^{(i_0)} = G^{(3)}$. Also the initial distribution is set to the uniform one $p_0^{(i)} = 1/8 \forall i$, since the graph structure is assumed to be completely unknown at the initial time $t = 0$. In this setting, we find in Fig. 2 (a) that the time evolution of $\{p_t^{(i)}\}$ according to Eq. (8) converges to the distribution with $p^{(3)} = p^{(5)} = 0.5$. That is, our scheme concludes the graph structure is $G^{(3)}$ or $G^{(5)}$, which means that it correctly identifies the chain-type structure of the network.

We now discuss about why the estimation is possible by measuring only a part of the network. For this purpose let us focus on the estimate (conditional expectation) of the z -component of the measured spin. In general, the continuous measurement of σ^z tends to increase the absolute value of z -component of the spin state [19], while the z -component of the first spin is diffused among the network due to the XY coupling Hamiltonian [28], [29]; i.e. *spin diffusion* occurs. Hence, intuitively, if the network is relatively “small” and the spin wave gets back to the measured spin quickly, the estimate of the z -component of the measured spin will change very fast, while in the opposite case the estimate will change slowly. We plot the time evolution of $Z_t^{(i_0)} = \text{Tr}(c\rho_t^{(i_0)})$ in Fig. 2 (b) and those of $Z_t^{(i)} = \text{Tr}(c\rho_t^{(i)})$ in Fig. 2 (c). These figures may support the validity of the above intuitive observation; that is, the chain structure is relatively a “large” network, and the corresponding estimates $Z_t^{(3)}$ and $Z_t^{(5)}$ are changing slowly. Moreover, Figs. 2 (b) and (c) indicate that the time evolutions of the nominal estimates $Z_t^{(3)}$ and $Z_t^{(5)}$ are similar to that of the true one $Z_t^{(i_0)}$, while the others are not. These facts clearly mean that the measurement even only on a part of the network certainly brings useful information for estimating the structure.

Remark: The algorithm (8) identifies the true network structure with high probability (in this case, it is over 80%), but it sometimes fails. For reliable estimation, therefore, we need to repeat the procedure and evaluate the averaged probability distribution $\{\langle p_t^{(i)} \rangle\}$.

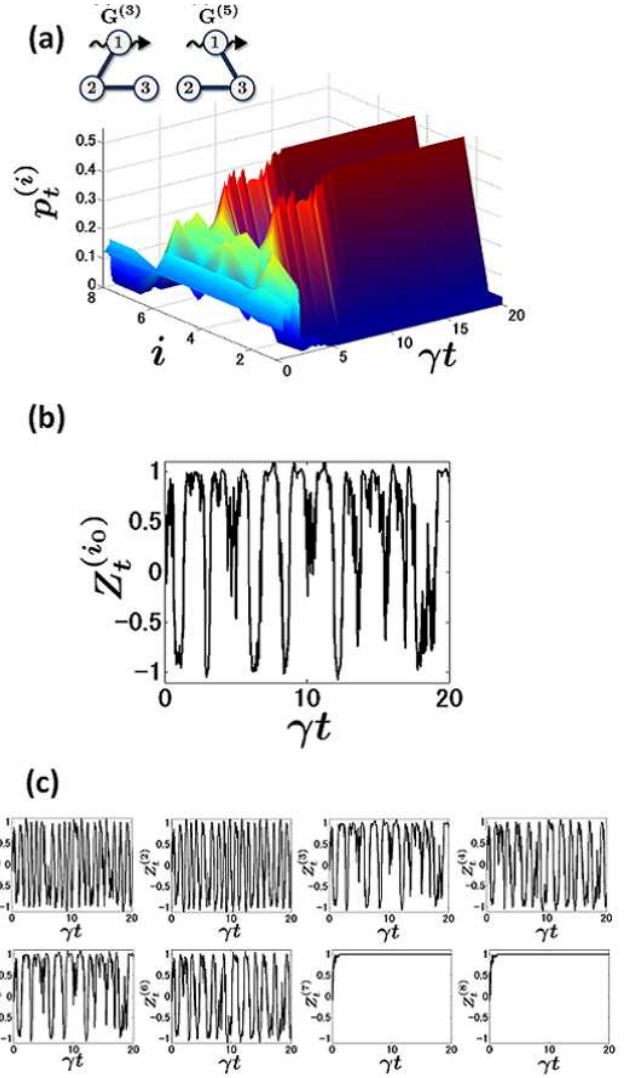


Fig. 2. Numerical simulation of Eq. (8) for the three-spin network with parameters $\lambda = \gamma$. The true graph structure is set to $G^{(i_0)} = G^{(3)}$. Figure (a) shows the time evolution of $\{p_t^{(i)}\}$, which converges to $p^{(3)} = p^{(5)} = 0.5$. Figures (b) and (c) show the conditional expectation of c of the true system with graph $G^{(i_0)} = G^{(3)}$ and those of all the nominal systems driven by Eq. (8), respectively.

C. Example 2: four spin case

We next consider the network composed of four spins, in which case there are totally $m = 64$ candidates for the graph structure. First, let us assume that the true graph is a chain depicted in Fig. 3 (a); the index of this graph is $i_0 = 27$. Figure 3 (a) shows the time evolution of $\langle p_t^{(i)} \rangle$ averaged over ten sample paths. The initial distribution is set to the uniform one $p_0^{(i)} = 1/64 \forall i$, because of the same reason explained in Sec. II-B. It is seen from the figure that $\langle p_t^{(i)} \rangle$ takes the maximum value at $i = 27, 29, 41, 45, 50, 51$, all of which correspond to the chain form. Hence, the estimator certainly identifies the true graph structure.

Figures 3 (b) and (c) illustrate the cases where the true graph structure is given by the star and the square, respectively; in both cases, the true structure is correctly identified.

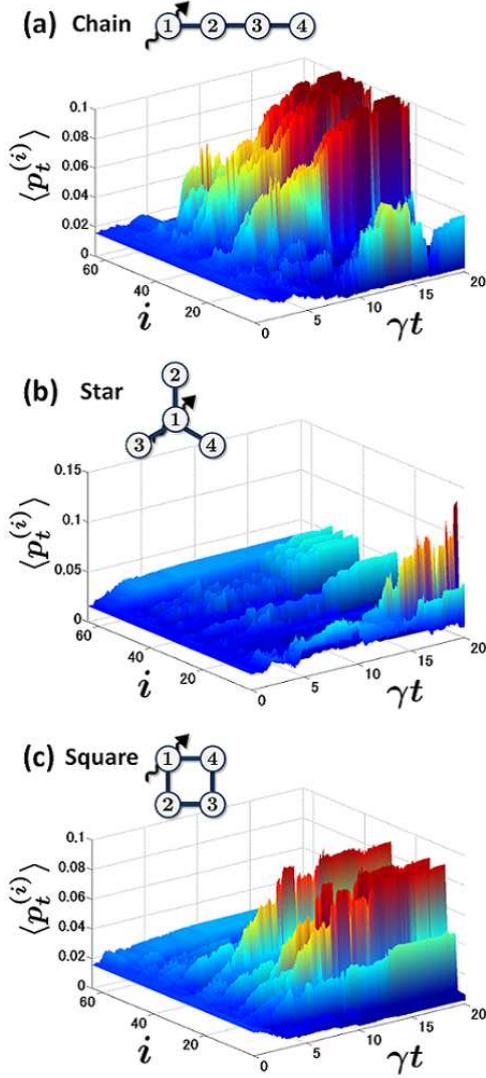


Fig. 3. Time evolution of the probability distribution $\langle p_t^{(i)} \rangle$ averaged over ten sample paths. The true graph structure is given by (a) chain (b) star, and (c) square. The parameter is set to $\lambda = \gamma$.

III. NETWORK INITIALIZATION VIA CONTINUOUS MEASUREMENT ON SINGLE NODE

Once the network structure has been identified, we can move forward to the stage of an appropriate state preparation. In this section, under the assumption that the network dynamics including its structure is completely known, we provide a scheme that deterministically stabilizes a spin coherent state $|\phi, \phi, \dots, \phi\rangle$ on the network, via continuous measurement performed only on a single spin. The deterministic driving of the state is brought from the mechanism of adaptive measurement; to make the idea clear, in this section, we first present an adaptive measurement method for state preparation of a single spin. Then it is applied to quantum networks with only a single node accessible.

A. Adaptive measurement for single spin state preparation

As indicated from Eq. (1), measuring a quantum system always brings a stochastic driving of the state. This means that only a fixed measurement does not help to stabilize a state deterministically at a certain target state. Combining measurement with feedback control is thus expected to overcome this issue, as actually demonstrated in several studies [18], [19], [20]. Adaptive measurement is a kind of feedback control, which does not introduce an additional actuator for control but rather changes the detector configuration based on the past measurement results. Thus a merit of adaptive measurement may appear in a practical situation where it costs cheaper than adding an actuator for feedback control. The applicability of this scheme for state preparation has been demonstrated in for instance [24], [25]; we here consider the same problem studied in these references and add a new result.

The problem of stabilizing a spin state via adaptive measurement is described as follows. Recall that the state under continuous measurement evolves in time according to Eq. (1). We then set $H = 0$ and assume that the coupling operator c can be changed in time as a function of ρ_t , so that ρ_t will deterministically converge to a desired target state. Without loss of generality, the target state can be set to $|0\rangle$. Then, it is reasonable to parameterize the coupling operator in the following form:

$$c_t = \begin{pmatrix} \cos \theta_t & e^{-i\delta_t} \sin \theta_t \\ e^{i\delta_t} \sin \theta_t & -\cos \theta_t \end{pmatrix}. \quad (9)$$

That is, the problem is to determine the dynamics of the parameters (θ_t, δ_t) so that the state ρ_t governed by the SME (1) with $H = 0$ and Eq. (9) will converge to $|0\rangle$.

The solution is obtained by introducing an appropriate cost function, as often done in control theory. Here we define the cost as

$$J_t = 1 - \text{Tr}(\sigma^z \rho_t).$$

This is non-negative and takes the minimum value 0 only when $\rho_t = |0\rangle\langle 0|$. Therefore, the parameters (θ_t, δ_t) should be chosen so that the cost will decrease in the mean sense; that is, if $d\mathbb{E}[J_t]/dt < 0$ is satisfied for any $\rho_t \neq |0\rangle\langle 0|$, the cost decreases until it reaches the minimum value 0, implying that $\rho_t \rightarrow |0\rangle\langle 0|$ almost surely. For more detailed discussion about this stochastic convergence, see [30]. To make a control law, let us parameterize the state as

$$\rho_t = \frac{1}{2} \begin{pmatrix} 1 + r_t \cos \alpha_t & r_t e^{-i\beta_t} \sin \alpha_t \\ r_t e^{i\beta_t} \sin \alpha_t & 1 - r_t \cos \alpha_t \end{pmatrix}. \quad (10)$$

Then, the derivative of $\mathbb{E}[J_t]$ is given by

$$\frac{d\mathbb{E}[J_t]}{dt} = -\frac{r_t}{2} \left[(\cos(2\theta_t) - 1) \cos \alpha_t + \sin(2\theta_t) \sin \alpha_t \cos(\delta_t - \beta_t) \right]. \quad (11)$$

Hence, when choosing the tuning parameters as

$$(\theta_t, \delta_t) = (\alpha_t/2, \beta_t), \quad (-\alpha_t/2, \beta_t + \pi), \quad (12)$$

we have

$$\frac{dE[J_t]}{dt} = -\frac{r_t}{2}(1 - \cos \alpha_t) \leq 0.$$

Then, from the result of stochastic stability [30], $dE[J_t]/dt \rightarrow 0$, thus equivalently $\alpha_t \rightarrow 0$ or $r_t \rightarrow 0$, is guaranteed. This means that after long time limit the state will be found on the positive half of the z axis in the Bloch sphere. But it is well known that an ideal continuous measurement of an observable always increases the purity of the conditional state; i.e., $r_t \rightarrow 1$. Combining these two results, we can conclude that the state converges to the target state $|0\rangle$ almost surely. The adaptive law (12) has been found in [25], though without rigorous proof. Hence here we present the result as a new contribution.

Theorem 1: The spin state subjected to the SME (1) with $H = 0$ and the adaptive measurement law (9), (10), and (12) converges to the target state $|0\rangle$ almost surely.

B. Network initialization via adaptive measurement on single node

Now we apply the adaptive measurement scheme developed in the previous subsection to N -spin quantum networks. We again consider the case where we are allowed to continuously measure only a single spin of the network. Also the network is driven by the XY coupling Hamiltonian as before. In this setting, the state under continuous measurement obeys the SME (1) with the following system matrices:

$$H = \sum_{(j,k) \in E(G)} \lambda(\sigma_j^x \otimes \sigma_k^x + \sigma_j^y \otimes \sigma_k^y), \quad (13)$$

$$c_t' = \begin{pmatrix} \cos \theta_t' & e^{-i\delta_t'} \sin \theta_t' \\ e^{i\delta_t'} \sin \theta_t' & -\cos \theta_t' \end{pmatrix} \otimes I_{2^{N-1}}. \quad (14)$$

The target state can be set to the spin coherent state $|0^{\otimes N}\rangle = |0, 0, \dots, 0\rangle$ without loss of generality, as in the single spin case. That is, the problem is to stabilize $|0^{\otimes N}\rangle$ in the network by applying continuous adaptive measurement performed only on the first spin.

Let us now determine the adaptive law of the parameters (θ_t', δ_t') of the measured observable (14). For this purpose, similar to the single spin case, we consider the following cost function:

$$J_t' = N - \text{Tr}(\sum_{j=1}^N \sigma_j^z \rho_t), \quad (15)$$

which is non-negative and takes the minimum value 0 only when $\rho_t = |0^{\otimes N}\rangle\langle 0^{\otimes N}|$. The time derivative of $E[J_t']$ is given by

$$\frac{dE[J_t']}{dt} = -\frac{r_t'}{2} \left[(\cos(2\theta_t') - 1) \cos \alpha_t' + \sin(2\theta_t') \sin \alpha_t' \cos(\delta_t' - \beta_t') \right], \quad (16)$$

where $(r_t', \alpha_t', \beta_t')$ are the parameters of the reduced quantum state $\rho_t' = \text{Tr}_{(2,3,4,\dots,N)}[\rho_t]$ as follows:

$$\rho_t' = \frac{1}{2} \begin{pmatrix} 1 + r_t' \cos \alpha_t' & r_t' e^{-i\beta_t'} \sin \alpha_t' \\ r_t' e^{i\beta_t'} \sin \alpha_t' & 1 - r_t' \cos \alpha_t' \end{pmatrix}. \quad (17)$$

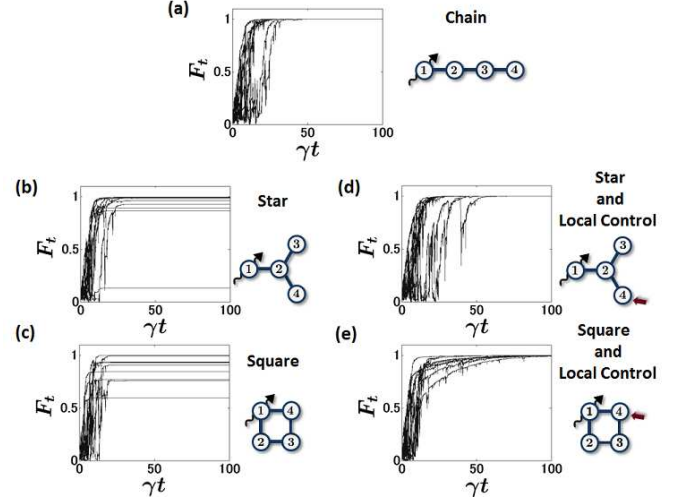


Fig. 4. Ten sample paths of the fidelity $F_t = \langle 0^{\otimes 4} | \rho_t | 0^{\otimes 4} \rangle$ for several network structures: (a) chain, (b,d) star, and (c,e) square. In (d,e) a local control indicated by the red arrow, which breaks permutation symmetry of the network, is introduced. The parameters are set to $\lambda = \gamma$.

The point here is that Eq. (16) has the same form as that for the single spin case, Eq. (11). Therefore, the adaptive law

$$(\theta_t', \delta_t') = (\alpha_t'/2, \beta_t'), \quad (-\alpha_t'/2, \beta_t' + \pi) \quad (18)$$

leads to $dE[J_t']/dt \rightarrow 0$ as before, which thus concludes $\rho_t' \rightarrow |0\rangle\langle 0|$. Also note that the measured observable (14) then becomes Eq. (3).

Unfortunately, the above result does not necessarily mean the convergence of the whole state ρ_t to the target. Nonetheless, now we know that the z -component of the first spin, which is continuously raised via the adaptive measurement, is diffused among the network due to the XY coupling Hamiltonian; hence the z -components of all spins will also increase. Furthermore, the target state $|0^{\otimes N}\rangle$ is a steady state of the SME with (13) and (3) (see Appendix). From these two facts, we expect that the state will converge to the target, if it is the unique steady state of the *controlled SME*, i.e., the SME driven by the adaptive measurement.

C. Numerical simulations and permutation symmetry

First, let us consider a four qubit network with chain graph whose first spin is continuously measured with the adaptive law (18). A notable feature of this network is that in this case $|0^{\otimes 4}\rangle$ is the unique steady state of the controlled SME, hence as mentioned above the deterministic stabilization would be expected. To evaluate the performance, we use the *fidelity* $F_t = \langle 0^{\otimes 4} | \rho_t | 0^{\otimes 4} \rangle$, which takes the maximum value 1 only when $\rho_t = |0^{\otimes 4}\rangle\langle 0^{\otimes 4}|$. Fig. 4 (a) displays ten sample paths of F_t , with the initial state $\rho_0 = (I_2/2)^{\otimes 4}$, and show that all trajectories converge to 1. Actually the same behavior was always observed in 300 simulations. Therefore it seems conclusive that the deterministic state stabilization is attained.

In the above example, a crucial property to realize the deterministic convergence is that the target is the unique steady

state of the controlled SME. Hence, we should study systems that do not have such uniqueness property. In particular, systems having *permutation symmetry* [14] are important; this property means that the Hamiltonian is invariant under the exchange of some specific pairs of spins. For example, the “star” formed network with Hamiltonian

$$H = \lambda(XXII + IXXI + IXIX + YYII + IYYI + IYIY), \quad (19)$$

where e.g. $XXII = \sigma^x \otimes \sigma^x \otimes I_2 \otimes I_2$, is permutation symmetry with respect to the exchange of the third and fourth spins. In this case, in addition to $|0^{\otimes 4}\rangle$, the “entangled state”

$$|\text{Ent}\rangle = |0, 0\rangle \otimes \frac{1}{\sqrt{2}}(|0, 1\rangle - |1, 0\rangle)$$

satisfies $H|\text{Ent}\rangle = 0$ and is a steady state of the SME due to the reason given in Appendix. $|\text{Ent}\rangle$ is a superposition of two states $|0, 0, 0, 1\rangle$ and $|0, 0, 1, 0\rangle$, which is invariant under the permutation operation. Now the state could go toward $|\text{Ent}\rangle$, hence the deterministic convergence to the target would not be expected. Actually, as seen in Fig. 4 (b), for the network with the star graph Hamiltonian (19), the state does not converge to the target deterministically; it converges to a mixed state on the subspace spanned by the steady states of the controlled SME. A similar result is observed in Fig. 4 (c) where the network with square-graph Hamiltonian is studied.

A reasonable method that circumvents the above issue would be the one adding a local control Hamiltonian such as $H_{\text{loc}} = IIIZ$, which breaks the permutation symmetry structure. In fact, with this local control Hamiltonian, we can attain the deterministic stabilization of the state at the target, both in the networks with star and square graphs, as shown in Fig. 4 (d) and (e), respectively.

IV. CONCLUSION

In this paper, we have provided particular continuous-measurement-based schematics for estimating and initializing spin networks that allow access only to a single node. As mentioned in Introduction, these are essential requirements for performing quantum information processing; in fact, if the structure of the network is identified and an appropriate initial state is prepared on it, as shown in [13], universal quantum computation on that network is possible via only a local control on a single spin.

APPENDIX

It is directly proven from [31], [32] that a pure state $|\phi\rangle$ is a steady state of the SME (1a) if and only if $|\phi\rangle$ is a common eigenvector of $iH + c^\dagger c/2$ and c . Clearly, $|0^{\otimes N}\rangle$ is an eigenvector of Eq. (3). Also noting the relation $(\sigma^x \otimes \sigma^x + \sigma^y \otimes \sigma^y)|00\rangle = 0$, we can readily see $H|0^{\otimes N}\rangle = 0$. Therefore, $|0^{\otimes N}\rangle$ is a common eigenvector of $iH + c^\dagger c/2$ and c , hence it is a steady state of the SME. Note that $|1^{\otimes N}\rangle$ is also a steady state, but the state driven by the adaptive measurement always converges to a state with its first node given by $|0\rangle$.

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